Triangles

2016

Very Short Answer Type Question [1 Mark]

Question 1.

If A ABC ~ ARPQ, AB = 3 cm, BC = 5 cm, AC = 6 cm, RP = 6 cm and PQ = 10 cm, then find QR

Solution:

$\Delta ABC \sim \Delta RPQ,$ (Given)

$\therefore \quad \frac{AB}{RP} = \frac{BC}{PQ} = \frac{AC}{RQ}$ (Proportional sides of similar triangles)

$$\frac{3}{6} = \frac{5}{10} = \frac{6}{QR}$$

$$\frac{1}{2} = \frac{1}{2} = \frac{6}{QR}$$

$\Rightarrow \quad QR = 12$ cm.

Short Answer Type Question I [2 Marks]

Question 2.

R and S are points on the sides DE and EF respectively of a ADEF such that ER = 5 cm, RD = 2.5 cm, SE = 1.5 cm and FS = 3.5 cm. Find whether RS || DF or not.

Solution:

Construction: Join RS

To find: RS || DF or not

Proof: We have RE = 5 cm

and RD = 2.5 cm

Now $\frac{RE}{RD} = \frac{5}{2.5} = \frac{2}{1}$

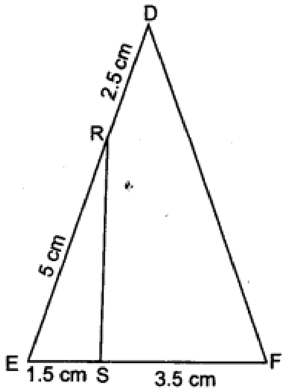
Similarly, we have, ES = 1.5 cm

and SF = 3.5 cm

Now, $\frac{SF}{ES} = \frac{3.5}{1.5} = \frac{7}{3}$

Here $\frac{RE}{RD} \neq \frac{SF}{ES}$

$\Rightarrow$ RS is not parallel to DF.



Short Answer Type Questions II [3 Marks]

Question 3.
From airport two aeroplanes start at the same time. If the speed of first aeroplane due North is 500 km/h and that of other due East is 650 km/h, then find the distance between two aeroplanes after 2 hours.

Solution:

Speed of aeroplane along north = 500 km/h
Speed of aeroplane along east = 650 km/h
Distance travelled by aeroplane in 2 hours in North direction.

$$= OB = 500 \times 2$$
$$= 1000 \text{ km.}$$

Distance travelled by aeroplane in 2 hours in East direction.

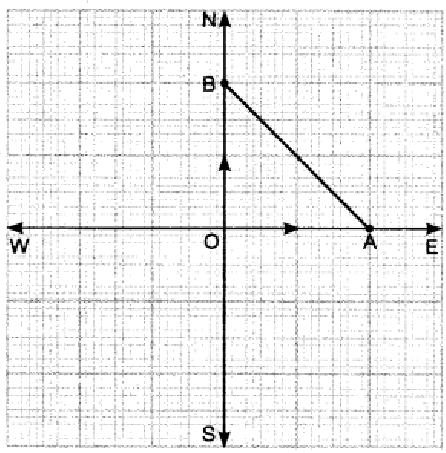
$$= OA = 650 \times 2$$
$$= 1300 \text{ km}$$

Distance between both aeroplanes after 2 hours = AB.

$\therefore$ $AB^2 = OB^2 + ON^2$

[By Pythagoras theorem in ΔAOB]

$$= (1000)^2 + (1300)^2$$
$$= 1000000 + 1690000$$
$$= 2690000$$
$$AB = 100\sqrt{269} \text{ km}$$

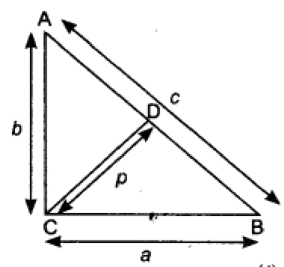


Question 4.
AABC, is right angled at C. If p is the length of the perpendicular from C to AB and a, b, c are the lengths of the sides opposite ∠A, ∠B, ∠C respectively then prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

Solution:

Given: In ΔACB, $\angle C = 90°$, and $CD \perp AB$.
Also, $AB = c$, $BC = a$, $CA = b$ and $CD = p$

To prove: $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$



Proof: In ΔABC, $\angle C = 90°$
Apply Pythagoras theorem

$$AB^2 = BC^2 + AC^2$$

$\Rightarrow$ $c^2 = a^2 + b^2$ ...(*i*)

Now, area of $\Delta ABC = \frac{1}{2} \times AB \times CD = \frac{1}{2} \times p \times c$ ...(*ii*)

Also area of $\Delta ABC = \frac{1}{2} \times AC \times BC = \frac{1}{2} \times b \times a$ ...(*iii*)

Equating (*iii*) and (*ii*), we have

$$\frac{1}{2}pc = \frac{1}{2}ab$$

$$pc = ab$$

$$c = \frac{ab}{p} \text{ and } c^2 = \frac{a^2b^2}{p^2}$$

Put value of c^2 in equation (*i*)

$$\frac{a^2b^2}{p^2} = a^2 + b^2$$

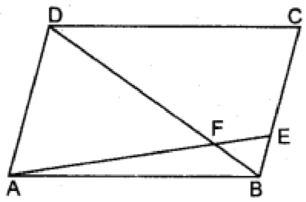
$$\frac{1}{p^2} = \frac{a^2}{a^2b^2} + \frac{b^2}{a^2b^2} \qquad \text{(dividing both sides by } a^2b^2\text{)}$$

$$\frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{a^2}$$

Question 5.

In the figure, ABCD is a parallelogram and E divides BC in the ratio 1: 3. DB and AE intersect at F. Show that DF = 4FB and AF = 4FE

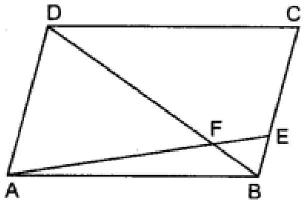
Solution:



Given: ABCD is a parallelogram and BE : EC : : 1 : 3

To show: DF = 4FB and AF = 4FE

Proof: In ΔADF and ΔEBF



$$\angle ADF = \angle EBF \quad \text{(Alternate angles)}$$

$$\angle AFD = \angle EFB \quad \text{(V.O.A.)}$$

$$\Delta ADF \sim \Delta EBF \quad \text{(by AA)}$$

$$\frac{DF}{BF} = \frac{AF}{EF} = \frac{AD}{BE} \quad ...(i)$$

as $\frac{BE}{EC} = \frac{1}{3} \Rightarrow EC = 3BE$

$\therefore$ BC = BE + CE = BE + 3BE

$\Rightarrow$ BC = 4BE

as AD = BC

$\therefore$ AD = 4BE

Put in (*i*), we get

$$\frac{DF}{BF} = \frac{AF}{EF} = \frac{4}{1}$$

DF = 4BF

and AF = 4EF

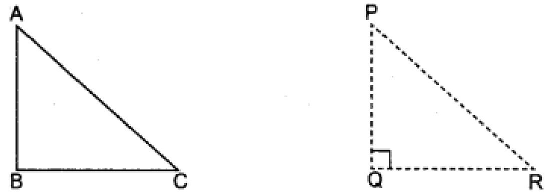
Long Answer Type Questions [4 Marks]

Question 6.

State and prove Converse of Pythagoras' Theorem.

Solution:

Statement: In a triangle, if the square of one side is equal to the sum of the squares of the other two sides then the angle opposite to the first side is right angle.



Given: In ΔABC, $AC^2 = AB^2 + BC^2$

To prove: $\angle ABC = 90°$

Construction: Construct a ΔPQR, such that $AB = PQ$ and $BC = QR$ and $\angle Q = 90°$

Proof: In ΔPQR, $\angle Q = 90°$ (Given)

$\therefore$ $PR^2 = PQ^2 + QR^2$ (By Pythagoras)

$PR^2 = AB^2 + BC^2$

(Using $PQ = AB$ and $QR = BC$ by construction) ...(*i*)

but $AC^2 = AB^2 + BC^2$ (Given) ...(*ii*)

Equating (*i*) and (*ii*) $PR^2 = AC^2$

$\Rightarrow$ $PR = AC$

Now, in ΔABC and ΔPQR

$AB = PQ$ (By construction)

$BC = QR$ (by construction)

$AC = PR$ (Proved)

$\therefore$ $\Delta ABC \cong \Delta PQR$ (By SSS)

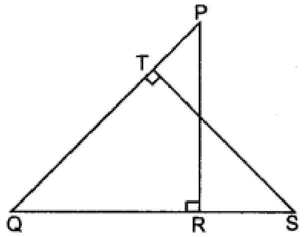
Hence $\angle B = \angle Q = 90°$

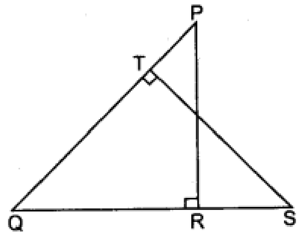
This shows that, ΔABC is an right-angled triangle.

Question 7.

In the figure, PQR and QST respectively. Prove that QR X QS= QP X QT

Solution:





Given: In ΔPQR, $\angle R = 90°$ and in ΔQTS, $\angle T = 90°$

To prove: $QR \times QS = QP \times QT$

Proof: In ΔPQR and ΔSQT

$\angle PQR = \angle SQT$ (Common)

$\angle PRQ = \angle STQ = 90°$ (Given)

$\Delta PQR \sim \angle SQT$ (By AA)

$\Rightarrow$ $\dfrac{QR}{QT} = \dfrac{QP}{QS}$

(Corresponding sides of similar triangles are proportion)

$\Rightarrow$ $QR \times QS = QP \times QT$

Very Short Answer Type Question [1 Mark]

Question 8.

In ADEW, AB || EW. If AD = 4 cm, DE = 12 cm and DW = 24 cm, then find the value of DB.

Solution:

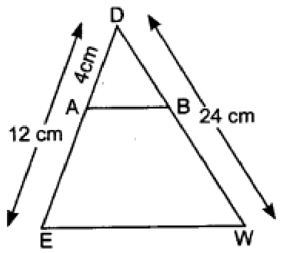
In ΔDEW, AB || EW

So, by B.P.T.,

$$\frac{AD}{DE} = \frac{DB}{DW}$$

$$\Rightarrow \quad \frac{4}{12} = \frac{DB}{24}$$

$$\Rightarrow \quad DB = 8 \text{ cm}$$



Short Answer Type Question I [2 Marks]

Question 9.

A ladder is placed against a wall such that its foot is at distance of 5 m from the wall and its top reaches a window 5/3 m above the ground. Find the length of the ladder

Solution:

$\because$ ABC is a right triangle, right angled at B.

So, by Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

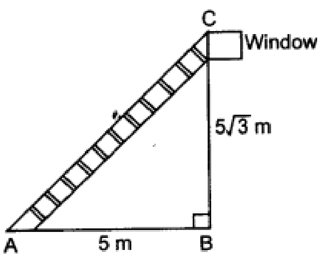
$$= 5^2 + (5\sqrt{3})^2 = 25 + 75$$

$$AC^2 = 100$$

$$\Rightarrow \quad AC = \sqrt{100}$$

$$\Rightarrow \quad AC = 10 \text{ m}$$

Hence, length of the ladder = 10 m.



Short Answer Type Questions II [3 Marks]

Question 10.

In figure, if ∠CAB = ∠CED, then prove that AB X DC = ED X BC.

Solution:

Given: $\angle CAB = \angle CED$

i.e. $\angle 1 = \angle 2$

To prove: $AB \times DC = ED \times BC$

Proof: In ΔCAB and ΔCED

$\angle 1 = \angle 2$ (Given)

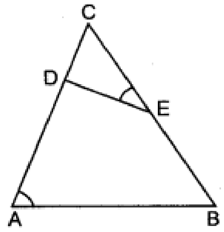
$\angle C = \angle C$ (Common)

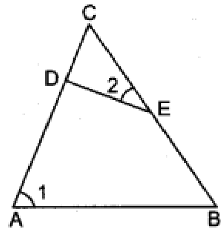
So, $\Delta CAB \sim \Delta CED$ (By AA similarity)

$$\Rightarrow \quad \frac{BC}{DC} = \frac{AB}{ED}$$

$$\Rightarrow \quad AB \times DC = BC \times ED$$

Hence Proved

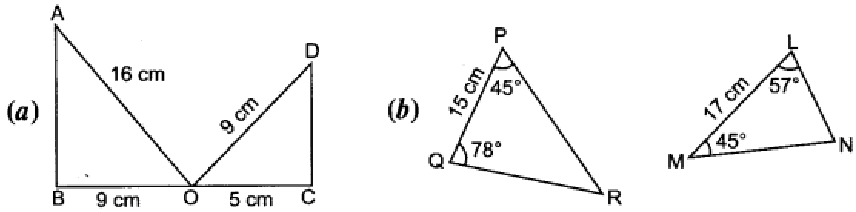




Question 11.

State whether the given pairs of triangles are similar or not. In mention the criterion.

Solution:



(*a*) In ΔABO, $\angle B = 90°$

By Pythagoras theorem,

$AB = \sqrt{16^2 - 9^2} = \sqrt{256 - 81} = \sqrt{175} = 5\sqrt{7}$ cm.

In ΔDCO, $\angle C = 90°$

So, by Pythagoras theorem,

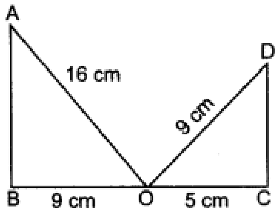
$DC = \sqrt{9^2 - 5^2} = \sqrt{81 - 25} = \sqrt{56} = 2\sqrt{14}$

$\therefore$ In ΔABO, $AB = 5\sqrt{7}$ cm, $OB = 9$ cm; $AO = 16$ cm

and In ΔDCO, $DC = 2\sqrt{14}$ cm; $OC = 5$ cm; $DO = 9$ cm

Clearly the sides of both the triangles are not proportional

So, given triangles are not similar.



(*b*)

In ΔPQR,	In ΔLMN
$\angle P + \angle Q + \angle R = 180°$	$\angle L + \angle M + \angle N = 180°$
$\Rightarrow 45° + 78° + \angle R = 180°$	$\Rightarrow 57° + 45° + \angle N = 180°$
$\Rightarrow \angle R = 180° - 123°$	$\Rightarrow \angle N = 180° - 102°$
$\Rightarrow \angle R = 57°$	$\Rightarrow \angle N = 78°$

Now, in ΔPQR and ΔLMN,

$\because \quad \angle P = \angle M = 45°$

$\angle Q = \angle N = 78°$

$\angle R = \angle L = 57°$

So, $\Delta PQR \sim \Delta MNL$ (By AAA criterion)

Long Answer Type Questions [4 Marks]

Question 12.

In ΔABC, from A and B altitudes AD and BE are drawn. Prove that $\Delta ADC \sim \Delta BEC$. Is ΔADB

~ ΔAEB and ΔADB ~ ΔADC?

Solution:

In ΔADC and ΔBEC,

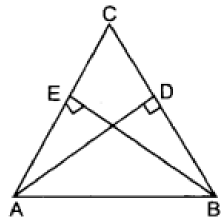
$\angle D = \angle E$ (Each 90°)

$\angle ACD = \angle BCE$ (Common)

So, $\Delta ADC \sim \Delta BCE$ (By AA similarity)

ΔADB is not similar to ΔAEB.

and ΔADB is not similar to ΔADC.



Question 13.

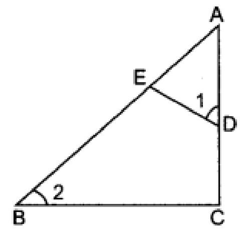
In ΔABC, if ∠ADE = ∠B, then prove that ΔADE ~ ΔABC. Also, if AD = 7.6 cm, AE = 7.2 cm, BE = 4.2 cm and BC = 8.4 cm, then find DE.

Solution:

Given: $\angle ADE = \angle B$, i.e. $\angle 1 = \angle 2$

To prove: $\Delta ADE \sim \Delta ABC$

Proof: In ΔADE and ΔABC



$\angle 1 = \angle 2$ [Given]

$\angle A = \angle A$ [Common]

So, $\Delta ADE \sim \Delta ABC$ [By AA similarity]

$$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \frac{7.6}{7.2+4.2} = \frac{DE}{8.4} \quad \{\because AB = AE + BE = 7.2 + 4.2\}$$

$$\Rightarrow \frac{7.6}{11.4} = \frac{DE}{8.4} \Rightarrow DE = \frac{7.6 \times 8.4}{11.4} = 5.6$$

Hence, DE = 5.6 cm.

2014

Very Short Answer Type Questions [1 Mark]

Question 14.

If ΔABC ~ ΔRPQ, AB = 3 cm, BC = 5 cm, AC = 6 cm, RP = 6 cm and PQ = 10 cm, then find QR.

Solution:

It is given that $\Delta ABC \sim \Delta RPQ$

$$\therefore \frac{AB}{RP} = \frac{AC}{QR}$$

Now, AB = 3 cm, RP = 6 cm, QR = ?, AC = 6 cm

$$\text{So, } \frac{3}{6} = \frac{6}{QR}$$

$$QR = \frac{36}{3} = 12 \text{ cm}$$

Question 15.

Let ΔABC ~ ΔDEF, ar (ΔABC) = 169 cm2 and ar (ΔDEF) = 121 cm^2. If AB = 26 cm^2 then find DE.

Solution:

It is given that $\Delta ABC \sim \Delta DEF$

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2}$$

[When two triangles are similar then the ratio of their areas is equal to ratio of squares of any of corresponding sides]

So,
$$\frac{169}{121} = \left(\frac{26}{DE}\right)^2$$
$$\frac{26}{DE} = \sqrt{\frac{169}{121}}$$
$$\frac{26}{DE} = \frac{13}{11}$$
$$DE = \frac{26 \times 11}{13}$$
$$DE = 22 \text{ cm}$$

Question 16.

If in an equilateral triangle, the length of the median is √3 cm, then find the length of the side of equilateral triangle.

Solution:

Let a be the side of equilateral triangle. Median is also the altitude in an equilateral triangle.

$$\therefore \quad (\text{Altitude})^2 + \left(\frac{a}{2}\right)^2 = (a)^2$$

$$(\sqrt{3})^2 + \frac{a^2}{4} = a^2 \qquad [\because \text{Altitude} = \text{Median} = \sqrt{3}\text{ cm}]$$

$$\therefore \quad \frac{12 + a^2}{4} = a^2$$
$$12 + a^2 = 4a^2$$
$$3a^2 = 12$$
$$a^2 = 4$$
$$a = 2$$

$\therefore$ Side of triangle = 2 cm

Short Answer Type Questions I [2 Marks]

Question 17.

In the figure, D andE are points on AB and AC respectively such that DE||BC. If AD = 1/3 BD and AE = 4.5 cm, find AC.

Solution:

DE || BC [Given]

$$AD = \frac{1}{3}BD$$
$$AE = 4.5 \text{ cm}$$

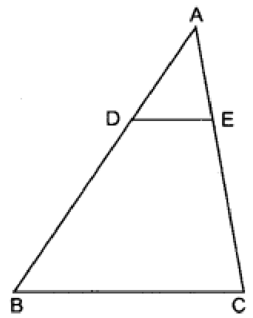
Now, $\because AD = \frac{1}{3}BD$

$$\therefore \quad \frac{AD}{BD} = \frac{1}{3} \qquad \ldots(i)$$

DE || BC,

$$\therefore \quad \frac{AD}{BD} = \frac{AE}{EC}$$

So,
$$\frac{1}{3} = \frac{4.5}{EC}$$
$$EC = 13.5 \text{ cm}$$
$$AC = AE + EC = 4.5 + 13.5 = 18 \text{ cm}$$



[By B.P.T.]

Question 18.

Determine whether the triangle having sides (a – 1) cm, 2 √a cm and (a + 1) cm is a right angled triangle.

Solution:

Given three sides of triangle are $(a-1)$ cm, $2\sqrt{a}$ cm and $(a+1)$ cm

In the given triangle, $(a+1)$ cm is the longest side.

Now, $(a+1)^2 = a^2 + 1 + 2a$ ...(*i*)

and sum of squares of other two sides $= (2\sqrt{a})^2 + (a-1)^2$

$= 4a + a^2 + 1 - 2a = a^2 + 1 + 2a$ ...(*ii*)

$\therefore$ From (*i*) and (*ii*)

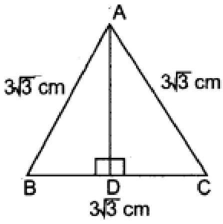
$$(a+1)^2 = (2\sqrt{a})^2 + (a-1)^2$$

$\therefore$ By converse of Pythagoras Theorem, given triangle is a right angled triangle.

Question 19.

In an equilateral triangle of side 3√3 cm, find the length of the altitude.

Solution:



In ΔABC, AD is altitude

AD is median also [$\because \Delta ABC$ is equilateral]

So, $BD = \frac{3\sqrt{3}}{2}$ cm

In ΔABD,

$$(AD)^2 + (BD)^2 = (AB)^2$$

$$\therefore \quad (AD)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2 = (3\sqrt{3})^2$$

$$AD^2 + \frac{27}{4} = 27$$

$$AD^2 = 27 - \frac{27}{4}$$

$$AD^2 = \frac{108-27}{4} \Rightarrow AD^2 = \frac{81}{4} \Rightarrow AD = \frac{9}{2} = 4.5 \text{ cm}$$

$$\therefore \quad \text{Altitude} = 4.5 \text{ cm}$$

Short Answer Type Questions II [3 Marks]

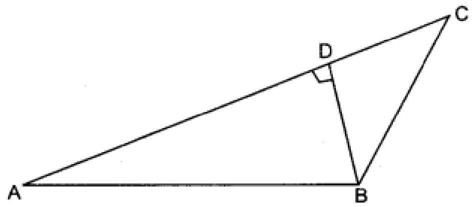
Question 20.

In the figure, ABC is a triangle and BD ⊥ AC. Prove that $AB^2 + CD^2 = AD^2 + BC^2$

Solution:

Given: A triangle ABC in which $BD \perp AC$.

To prove: $AB^2 + CD^2 = AD^2 + BC^2$



Proof: In right ΔADB,

$AB^2 = AD^2 + BD^2$ [By Pythagoras Theorem] ...(*i*)

In right ΔCDB,

$BC^2 = CD^2 + BD^2$ [By Pythagoras Theorem] ...(*ii*)

Subtracting equation (*ii*) from (*i*), we get,

$$AB^2 - BC^2 = AD^2 + BD^2 - CD^2 - BD^2$$

$$AB^2 - BC^2 = AD^2 - CD^2$$

$$\therefore \quad AB^2 + CD^2 = AD^2 + BC^2$$

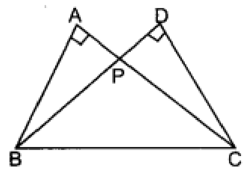
Hence proved.

Question 21.

Right angled triangles BAC and BDC are right angled at A and D and they are on same side of BC. If AC and BD intersect at P, then prove that AP x PC = PB X DP.

Solution:

In ΔAPB and ΔDPC,



$\angle BAP = \angle CDP = 90°$ [Given]

$\angle APB = \angle DPC$ [Vertically opposite angles]

$\therefore \quad \Delta APB \sim \Delta DPC$ [by AA similarity of triangles.]

$$\therefore \quad \frac{AP}{DP} = \frac{PB}{PC}$$

$$AP \times PC = PB \times DP$$

Hence proved.

Question 22.

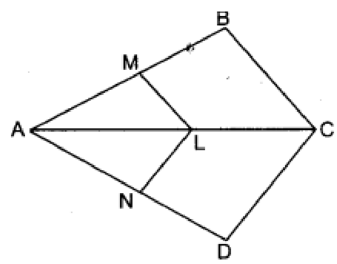
In the given figure, if LM || CB and LN|| CD, prove that AM X AD = AB X AN.

Solution:

Given: $LM \parallel CB$

$LN \parallel CD$

To prove: $AM \times AD = AB \times AN$



Proof: In ΔBAC,

$LM \parallel BC$

$$\therefore \quad \frac{AM}{MB} = \frac{AL}{LC} \quad \text{...(i) [By Thales Theorem]}$$

In ΔDAC,

$LN \parallel CD$

$$\therefore \quad \frac{AN}{ND} = \frac{AL}{LC} \quad \text{...(ii) [By Thales Theorem]}$$

$\therefore$ From (*i*) and (*ii*)

$$\frac{AM}{MB} = \frac{AN}{ND}$$

or

$$\frac{MB}{AM} = \frac{ND}{AN}$$

$$\frac{MB}{AM} + 1 = \frac{ND}{AN} + 1 \quad \text{[Adding 1 on both sides]}$$

$$\frac{MB + AM}{AM} = \frac{ND + AN}{AN}$$

$$\frac{AB}{AM} = \frac{AD}{AN}$$

$$\therefore \quad AM \times AD = AB \times AN$$

Hence proved.

Question 23.

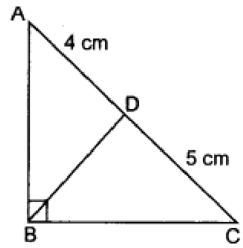
In the given figure, ABC is a triangle, right angled at B and $BD \perp AC$. If AD = 4 cm and CD = 5 cm, find BD and AB.

Solution:

and CD = 5 cm, find BD and AB.

Given: A right ΔABC in which $BD \perp AC$.

$$AD = 4 \text{ cm}, CD = 5 \text{ cm}$$



In ΔABC,

$$AB^2 + BC^2 = AC^2$$

$$AB^2 + BC^2 = (4 + 5)^2 = (9)^2 = 81 \quad \text{...(i) [By Pythagoras Theorem]}$$

In ΔADB,

$$AD^2 + BD^2 = AB^2 \quad \text{...(ii) [By Pythagoras Theorem]}$$

In ΔCDB,

$$CD^2 + BD^2 = BC^2 \quad \text{...(iii) [By Pythagoras Theorem]}$$

Adding equations (*ii*) and (*iii*), we get

$$AD^2 + CD^2 + 2BD^2 = AB^2 + BC^2$$

$$(4)^2 + (5)^2 + 2BD^2 = 81 \quad \text{[From equation (i)]}$$

$$2BD^2 + 16 + 25 = 81$$

$$2BD^2 = 81 - 41$$

$$2BD^2 = 40$$

$$BD^2 = 20$$

$$BD = \sqrt{20} = 2\sqrt{5} \text{ cm}$$

Putting, $BD = 2\sqrt{5}$ and $AD = 4$ cm in equation (*ii*), we get

$$(4)^2 + (2\sqrt{5})^2 = AB^2$$

$$AB^2 = 16 + 20$$

$$AB^2 = 36$$

$$AB = \sqrt{36} = 6 \text{ cm}$$

Question 24.

Equiangular triangles are drawn on sides of right angled triangle in which perpendicular is double of its base. Show that area of triangle on the hypotenuse is the sum of areas of the other two triangles?

Solution:

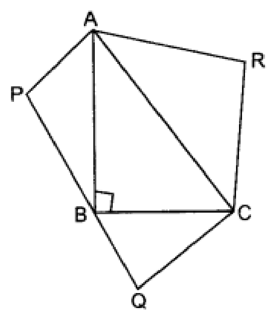
Given: A right angled triangle ABC with right angled at B.

Equiangular triangles PAB, QBC and RAC are described on sides AB, BC and CA respectively.

Let $BC = x$ and $AB = 2x$

To prove: $ar(\Delta PAB) + ar(\Delta QBC) = ar(\Delta RAC)$

Proof: $\because$ Equiangular triangles are equilateral also,



$$\therefore \quad \text{Area of } \Delta PAB = \frac{\sqrt{3}}{4} \times (2x)^2$$

$$= \sqrt{3}x^2 \quad \text{...(i)}$$

$$\text{Area of } \Delta QBC = \frac{\sqrt{3}}{4} \times (x)^2$$

$$= \frac{\sqrt{3}}{4}x^2 \quad \text{...(ii)}$$

$$\text{Area of } \Delta RAC = \frac{\sqrt{3}}{4} \times (AC)^2$$

$$= \frac{\sqrt{3}}{4} \times 5x^2 \quad [\because \text{In } \Delta ABC, AB^2 + BC^2 = AC^2$$

$$AC^2 = (2x)^2 + x^2 = 5x^2]$$

$$= \frac{5\sqrt{3}x^2}{4} \quad \text{...(iii)}$$

Adding (*i*) and (*ii*)

$$ar(\Delta PAB) + ar(\Delta QBC) = \sqrt{3}x^2 + \frac{\sqrt{3}x^2}{4} = \frac{4\sqrt{3}x^2 + \sqrt{3}x^2}{4} = \frac{5\sqrt{3}x^2}{4}$$

$$= ar(\Delta RAC) \quad \text{[From (iii)]}$$

$$\therefore \quad ar(\Delta PAB) + ar(\Delta QBC) = ar(\Delta RAC)$$

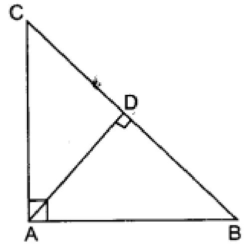
Hence proved.

Question 25.

If in a right angle AABC, right angled at A, AD $\perp$ BC, then prove that $AB^2 + CD^2 = BD^2 +$

AC^2

Solution:



In right ΔADB, $\angle D = 90°$

$AB^2 = AD^2 + BD^2$ ...(*i*) [By Pythagoras Theorem]

In right ΔADC, $\angle D = 90°$

$AC^2 = AD^2 + CD^2$ ...(*ii*) [By Pythagoras Theorem]

Subtracting equation (*ii*) from (*i*), we get

$AB^2 - AC^2 = AD^2 + BD^2 - AD^2 - CD^2$

$AB^2 + CD^2 = BD^2 + AC^2$ Hence proved.

Long Answer Type Questions [4 Marks]

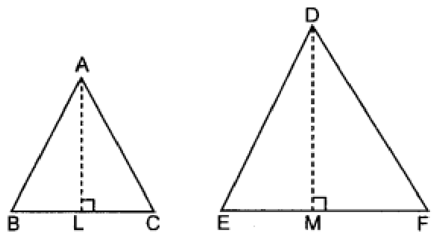
Question 26.

Prove that the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

Solution:

Given: Two triangles ABC and DEF such that $\Delta ABC \sim \Delta DEF$.

To prove: $\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{AB^2}{DE^2} = \dfrac{BC^2}{EF^2} = \dfrac{AC^2}{DF^2}$



Construction: Draw $AL \perp BC$ and $DM \perp EF$

Proof: In ΔALB and ΔDME,

$\angle ALB = \angle DME = 90°$ [By Construction]

$\angle B = \angle E$ [$\because \Delta ABC \sim \Delta DEF$]

$\therefore$ By AA criterion of similarity,

$\Delta ALB \sim \Delta DME$

So, $\dfrac{AL}{DM} = \dfrac{AB}{DE}$ ...(*i*)

$\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$ ...(*ii*) [Ratio of corresponding sides of similar triangles are equal]

From (*i*) and (*ii*), we get

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AL}{DM} \quad ...(iii)$$

Now, $\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{\frac{1}{2}(BC \times AL)}{\frac{1}{2}(EF \times DM)}$

$\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{BC}{EF} \times \dfrac{AL}{DM}$

$\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{BC \times BC}{EF \times EF} = \dfrac{BC^2}{EF^2}$ [From (*iii*)]

And $\dfrac{BC^2}{EF^2} = \dfrac{AB^2}{DE^2} = \dfrac{AC^2}{DF^2}$ [From (*iii*)]

Hence, $\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{AB^2}{DE^2} = \dfrac{BC^2}{EF^2} = \dfrac{AC^2}{DF^2}$ Hence proved.

Question 27.

In ΔABC, $AX \perp BC$ and Y is middle point of BC. Then prove that,

(*i*) $\mathbf{AB^2 = AY^2 + \dfrac{BC^2}{4} - BC.XY}$

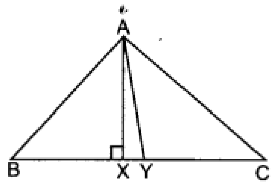
(*ii*) $\mathbf{AC^2 = AY^2 + \dfrac{BC^2}{4} + BC.XY}$

Solution:

In ΔABC, $AX \perp BC$ and Y is middle point of BC.
Then prove that,

(i) $AB^2 = AY^2 + \frac{BC^2}{4} - BC.XY$

(ii) $AC^2 = AY^2 + \frac{BC^2}{4} + BC.XY$



Given: A ΔABC in which $AX \perp BC$ and Y is mid-point of BC.

To prove: (i) $AB^2 = AY^2 + \frac{BC^2}{4} - BC.XY$

(ii) $AC^2 = AY^2 + \frac{BC^2}{4} + BC.XY$

Proof: (i) In ΔABX.

$AB^2 = AX^2 + BX^2$ [By Pythagoras Theorem]

$AB^2 = AX^2 + (BY - XY)^2$

$AB^2 = AX^2 + \left(\frac{BC}{2} - XY\right)^2$ [$\because$ Y is mid point of BC]

$AB^2 = AX^2 + \frac{BC^2}{4} + XY^2 - 2\left(\frac{BC}{2}\right)(XY)$

$AB^2 = (AX^2 + XY^2) + \frac{BC^2}{4} - \frac{2BC}{2}.XY$

$AB^2 = AY^2 + \frac{BC^2}{4} - BC.XY$ [$\because$ In ΔAXY, $AX^2 + XY^2 = AY^2$]

Hence proved.

(ii) In ΔAXC,

$AC^2 = AX^2 + XC^2$ [By Pythagoras Theorem]

$AC^2 = AX^2 + (XY + YC)^2$

$AC^2 = AX^2 + \left(XY + \frac{BC}{2}\right)^2$ [$\because$ In Y is mid-point of BC]

$AC^2 = (AX^2 + XY^2) + \frac{BC^2}{4} + 2(XY).\left(\frac{BC}{2}\right)$

$AC^2 = AY^2 + \frac{BC^2}{4} + BC.XY$ [$\because$ In ΔAXY, $AX^2 + XY^2 = AY^2$]

Hence proved.

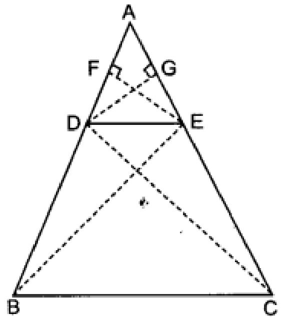
Question 28.

Prove that if a line is drawn parallel to one side of a triangle to intersect the othertwo sides at distinct points, then other two sides are divided in the same ratio.

Solution:

Given: A triangle ABC in which DE || BC and DE intersects AB in D and AC in E.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$



Construction: Join BE, CD and draw $EF \perp BA$ and $DG \perp CA$

Proof: $\because$ EF is perpendicular to AB.

$\therefore$ EF is height of triangles ADE and DBE.

Now, $$\frac{ar(\Delta ADE)}{ar(\Delta DBE)} = \frac{\frac{1}{2}(AD \times EF)}{\frac{1}{2}(DB \times EF)} = \frac{AD}{DB} \quad ...(i)$$

Similarly, $DG \perp CA$, so DG is height of ΔADE and ΔDEC.

$$\therefore \qquad \frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{\frac{1}{2}(AE \times DG)}{\frac{1}{2}(EC \times DG)} = \frac{AE}{EC} \qquad ...(ii)$$

But, ΔDBE and ΔDEC are on same base DE and between same parallels DE and BC.

$$\therefore \qquad ar(\Delta DBE) = ar(\Delta DEC)$$

$$\frac{1}{ar(\Delta DBE)} = \frac{1}{ar(\Delta DEC)}$$

Multiplying both sides by ar (ΔADE)

$$\frac{ar(\Delta ADE)}{ar(\Delta DBE)} = \frac{ar(\Delta ADE)}{ar(\Delta DBC)}$$

Thus, $$\frac{AD}{DB} = \frac{AE}{EC} \qquad \text{[From } (i) \text{ and } (ii)]$$

Question 29.

Prove that the sum of square of the sides of a rhombus is equal to the sum of squares of its diagonals.

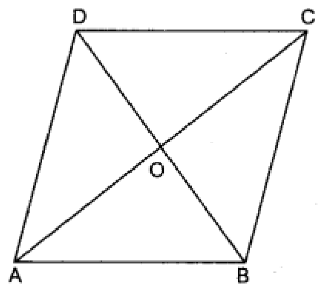
Solution:

Given: ABCD is a rhombus in which diagonals AC and BD intersect at O.

To prove: $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

Proof: As diagonals of a rhombus intersect/bisect each other at 90°.

$\therefore$ ΔAOD, ΔBOA, ΔCOB and ΔDOC are right triangles



and $$OD = OB = \frac{BD}{2} \qquad ...(i)$$

$$OA = OC = \frac{AC}{2} \qquad ...(ii)$$

Now by Pythagoras Theorem,

In ΔAOD,

$$AD^2 = OD^2 + OA^2 \qquad ...(iii)$$

In ΔBOA,

$$AB^2 = OA^2 + OB^2 \qquad ...(iv)$$

In ΔCOB,

$$BC^2 = OC^2 + OB^2 \qquad ...(v)$$

In ΔDOC,

$$CD^2 = OC^2 + OD^2 \qquad ...(vi)$$

Adding equations (*iii*), (*iv*), (*v*) and (*vi*), we get

$$AD^2 + AB^2 + BC^2 + CD^2 = 2OD^2 + 2OA^2 + 2OB^2 + 2OC^2$$

$$AB^2 + BC^2 + CD^2 + AD^2 = 2(OD^2 + OA^2 + OB^2 + OC^2)$$

$$AB^2 + BC^2 + CD^2 + AD^2 = 2\left[\left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2\right]$$

$$AB^2 + BC^2 + CD^2 + AD^2 = 2\left[\frac{2(BD^2 + AC^2)}{4}\right]$$

$$\therefore AB^2 + BC^2 + CD^2 + AD^2 = BD^2 + AC^2$$

Hence proved.

Question 30.

In AABC, X is any point on AC. If Y, Z , U and Y are the middle points of AX, XC, AB and BC respectively, then prove that UY || VZ and UV|| YZ.

Solution:

Given: In ΔABC, Y, Z, U and V are mid-points of AX, XC, AB and BC.

To prove: UY || VZ and UV || YZ

Construction: Join BX

Proof: In ΔABX,

$$AU = UB, \text{ i.e. } \frac{AU}{UB} = \frac{1}{1}$$

and $$AY = YX, \text{ i.e. } \frac{AY}{YX} = \frac{1}{1}$$

$$\therefore \quad \frac{AU}{UB} = \frac{AY}{YX}$$

So, UY || BX

Similarly, in ΔBCX,

$$BX \parallel VZ$$

$\therefore$ From (*i*) and (*ii*)

$$UY \parallel VZ$$

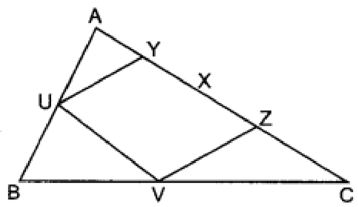
In ΔABC,

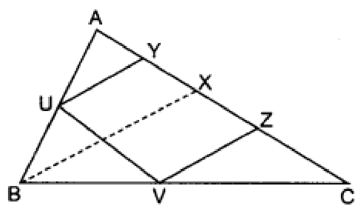
$$\frac{BU}{UA} = \frac{BV}{VC} = \frac{1}{1}$$

$\therefore$ By converse of B.P.T.,

$$UV \parallel YZ$$

From equation (*iii*) and (*iv*), UY || VZ and UV || YZ.





Question 31.

In ΔABC, ZB = 90°, BD $\perp$ AC, ar (ΔABC) = A and BC = a, then prove that

$$\mathbf{BD} = \frac{2Aa}{\sqrt{4A^2 + a^4}}$$

Solution:

In ΔABC, $\angle$B = 90°, BD $\perp$ AC, ar (ΔABC) = A and BC = *a*, then prove that

$$\mathbf{BD} = \frac{2Aa}{\sqrt{4A^2 + a^4}}$$

Given: Area of ΔABC = A

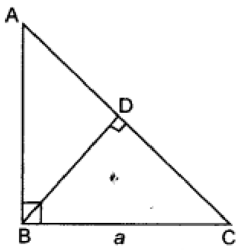
BC = a and BD $\perp$ AC

To prove: $$BD = \frac{2Aa}{\sqrt{4A^2 + a^4}}$$

Proof: A = Area of ΔABC

$$A = \frac{1}{2} BC \times AB = \frac{1}{2} \times a \times AB$$

$$AB = \frac{2A}{a} \qquad \text{...}(i)$$



In ΔADB and ΔABC

$\angle$ADB = $\angle$ABC = 90°

$\angle$BAD = $\angle$BAC [Common]

$\therefore$ By AA similarity criterion, $\Delta ADB \sim \Delta ABC$

$$\therefore \quad \frac{AB}{AC} = \frac{BD}{BC} \qquad \text{...}(ii)$$

In ΔABC, $AB^2 + BC^2 = AC^2$ [By Pythagoras Theorem]

$$\frac{4A^2}{a^2} + a^2 = AC^2$$

$$AC = \sqrt{\frac{4A^2 + a^4}{a^2}} = \frac{\sqrt{4A^2 + a^4}}{a}$$

From (i) and (ii), we get

$$\frac{2A}{a \times AC} = \frac{BD}{a}$$

$$BD = \frac{2A}{AC} = \frac{2Aa}{\sqrt{4A^2 + a^4}}$$

Hence proved.

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Short Answer Type Questions I [2 Marks]

Question 32.

In the figure, PQR and SQR are two right triangles with common hypotenuse QR. If PR and SQ intersect at M such that PM = 3 cm, MR = 6 cm and SM = 4 cm, find the length of MQ.

Solution:

Consider the triangles MPQ and MSR,

$\angle P = \angle S$ (Each 90°)

$\angle 1 = \angle 2$ (Vertically opposite angles)

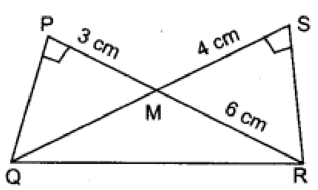
So, $\Delta MPQ \sim \Delta MSR$ (By AA similarity)

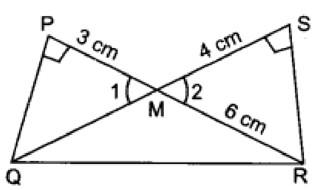
$$\frac{PM}{SM} = \frac{MQ}{MR}$$

$$\frac{3}{4} = \frac{MQ}{6}$$

$$MQ = \frac{3}{4} \times 6 = \frac{9}{2} = 4.5 \text{ cm}$$

Hence, MQ = 4.5 cm.





Question 33.

Find the length of each altitude of an equilateral triangle

Solution:

ΔABC is an equilateral triangle.

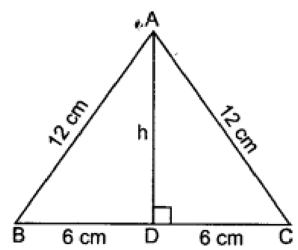
$\because$ AD is the altitude of height 'h' (say) then AD is median of BC also.

$\Rightarrow \quad CD = BD = 6$ cm.

In right ΔADC, $\angle D = 90°$

$$AC^2 = AD^2 + CD^2 \qquad \text{(By Pythagoras Theorem)}$$

$$12^2 = h^2 + 6^2$$



$$h^2 = 12^2 - 6^2$$
$$= 144 - 36 = 108$$
$$h = \sqrt{108} \Rightarrow h = 6\sqrt{3} \text{ cm.}$$

Hence, length of altitude is $6\sqrt{3}$ cm.

Short Answer Type Questions II [3 Marks]

Question 34.

In the figure, DB⊥BC, DE⊥AB and AC⊥ BC prove that

$$\frac{BE}{DE} = \frac{AC}{BC}$$

Solution:

Given DB ⊥ BC, i.e. $\angle 1 + \angle 2 = 90°$

DE ⊥ AB, i.e. $\angle E = 90°$

AC ⊥ BC, i.e. $\angle C = 90°$

To prove: $\frac{BE}{DE} = \frac{AC}{BC}$

Proof: In ΔABC, $\angle C = 90°$

$$\angle A + \angle 2 = 90° \quad \text{...}(i)$$

Also, $$\angle 1 + \angle 2 = 90° \text{(given)} \quad \text{...}(ii)$$

From (*i*) and (*ii*), we get

$$\angle 1 + \angle 2 = \angle 2 + \angle A$$

$$\Rightarrow \quad \angle 1 = \angle A$$

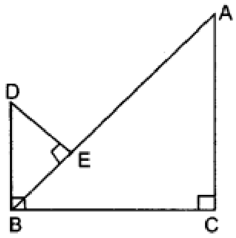
Now, in ΔDEB and ΔBCA

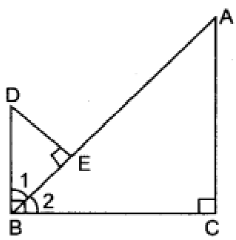
$$\angle E = \angle C \quad \text{(Each 90°)}$$

$$\angle 1 = \angle A \quad \text{(Proved above)}$$

$$\therefore \quad \Delta DEB \sim \Delta BCA$$

$$\Rightarrow \quad \frac{BE}{AC} = \frac{DE}{BC} \Rightarrow \frac{BE}{DE} = \frac{AC}{BC}$$





Question 35.

In the given figure, ABCD is a rectangle. P is the mid-point of DC. If QB = 7 m, AD = 9 cm and DC = 24 cm, then prove that ∠APQ = 90°

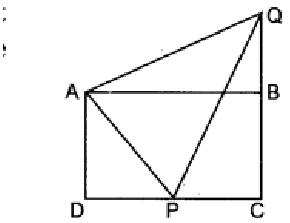
Solution:

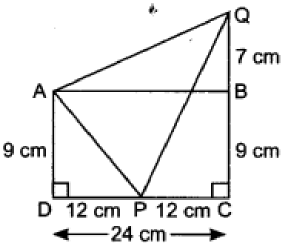
Given: ABCD is a rectangle.
P is the mid-point of DC, i.e. DP = PC
BQ = 7 cm, AD = 9 cm; CD = 24 cm
To prove: $\angle APQ = 90^\circ$
Proof. In ΔQCP, $\angle C = 90^\circ$

$$PQ^2 = PC^2 + QC^2 \quad \text{(By Pythagoras theorem)}$$
$$= 12^2 + 16^2 = 144 + 256 = 400$$
$$\Rightarrow \quad PQ = 20 \text{ cm}$$

Now, in ΔADP, $D = 90^\circ$

$$\text{So,} \quad AP^2 = AD^2 + DP^2 \quad \text{(By Pythagoras theorem)}$$





$$= 9^2 + 12^2 = 81 + 144 = 225$$
$$\Rightarrow \quad AP = 15 \text{ cm}$$

Now, in ΔQBA, $B = 90^\circ$

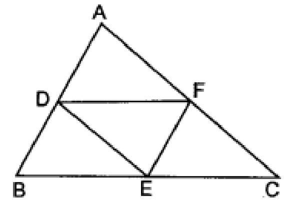
$$\Rightarrow \quad AQ^2 = AB^2 + QB^2 \quad \text{(By Pythagoras theorem)}$$
$$= 24^2 + 7^2 = 576 + 49 = 625$$
$$\Rightarrow \quad AQ = 25 \text{ cm}$$
$$\because \quad 625 = 400 + 225$$
$$\Rightarrow \quad AQ^2 = PQ^2 + AP^2$$
$$\Rightarrow \quad \angle APQ = 90^\circ \quad \text{(By converse of Pythagoras theorem)}$$

Question 36.

In AABC, D, E and F are the mid-points of AB, BC and AC respectively. Find the ratio of area of ΔABC and area of ΔDEF.

Solution:

Given: D, E and F are the mid-points of AB, BC and AC respectively of ΔABC.



Since, D and F are the mid-points of AB and AC respectively

So, by mid-point theorem, $DF = \frac{1}{2}BC$ and $DF || BC$

$\Rightarrow \quad DF = BE$ and $DF || BE$

Similarly, $EF = BD$ and $EF || BD$

$\Rightarrow$ BEFD is a parallelogram

So, $\Delta BDE \cong \Delta FED$ ...(*i*) [Diagonal divides a parallelogram ino two congruent triangles]

Similarly,

ECFD and ADEF are also a parallelogram

$\Rightarrow \quad \Delta ECF \cong \Delta FDE$ ...(*ii*) and $\Delta FED \cong \Delta DAF$ ...(*iii*)

From (*i*), (*ii*) and (*iii*)

$\Delta BDE \cong \Delta FED \cong \Delta EFC \cong \Delta DAF.$

We know that congruent triangles have equal areas.

So, ar (ΔBDE) = ar (ΔFED) = ar(ΔEFC) = ar(ΔDAF)

Let ar $(\Delta FED))$ = x sq units then ar (ΔABC) = $4x$ sq units.

Now, $\angle A = \angle E$ [Opposite angles of parallelogram ADEF are equal]

$\angle B = \angle F$ [Opposite angle of parallelogram BEFD are equal]

$\Rightarrow \quad \Delta ABC \sim \Delta EFD$ [By AA similarity]

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{4x}{x} = \frac{4}{1}$$

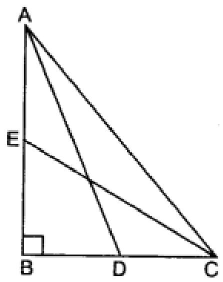
$ar(\Delta ABC) : ar(\Delta DEF) = 4 : 1$

Long Answer Type Questions [4 Marks]

Question 37.

In the figure, ABC is a right triangle, right angled at B. AD and CF are two medians drawn from A and C respectively. If AC = 5 cm and AD = 3√5/2cm. Find the length of CE.

Solution:



In right-triangle ABD, $\angle B = 90°$

$\therefore \quad AD^2 = AB^2 + BD^2$ [By Pythagoras Theorem]

$\Rightarrow \quad AD^2 = AB^2 + \left(\frac{BC}{2}\right)^2$ [$\because$ BD = DC]

$\Rightarrow \quad AD^2 = AB^2 + \frac{BC^2}{4}$ ...(*i*)

Now, in right-triangle EBC, $\angle B = 90°$,

$\therefore \quad CE^2 = BC^2 + BE^2$ [By Pythagoras Theorem]

$\Rightarrow \quad CE^2 = BC^2 + \left(\frac{AB}{2}\right)^2$ [$\because$ BE = AE]

$\Rightarrow \quad CE^2 = BC^2 + \frac{AB^2}{4}$ ...(*ii*)

Adding (*i*) and (*ii*), we get

$$AD^2 + CE^2 = AB^2 + \frac{BC^2}{4} + BC^2 + \frac{AB^2}{4}$$

$$AD^2 + CE^2 = \frac{5}{4}(AB^2 + BC^2)$$

$$AD^2 + CE^2 = \frac{5}{4}AC^2$$ [$\because$ By Pythagoras Theorem in ΔABC $AC^2 = AB^2 + BC^2$]

$$\left(\frac{3\sqrt{5}}{2}\right)^2 + CE^2 = \frac{5}{4} \times 25$$

$$CE^2 = \frac{125}{4} - \frac{45}{4}$$

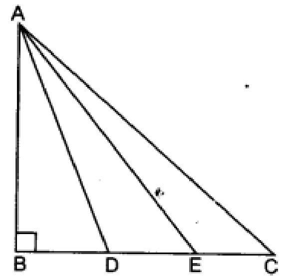
$$CE^2 = 20$$

$$\therefore \quad CE = \sqrt{20} = 2\sqrt{5} \text{ cm}$$

Question 38.

ABC is a right triangle, right-angled at B. D and E trisect BC, prove that $8AE^2 = 3AC^2 + 5AD^2$

Solution:



Given: A right-triangle ABC right angled at B.

Points D and E trisect BC. i.e. BD = DE = EC

To prove: $8AE^2 = 3AC^2 + 5AD^2$

Proof: Let $BD = x$

and $BD = DE = EC$ [Given]

$\therefore$ $BD = DE = EC = x$.

Now, in ΔABD.

$AD^2 = AB^2 + BD^2$ [By Pythagoras Theorem]

$AD^2 = AB^2 + x^2$ ...(*i*)

In ΔABE

$AE^2 = AB^2 + BE^2$ [By Pythagoras Theorem]

$AE^2 = AB^2 + 4x^2$ ...(*ii*) [$\because BE = BD + DE = x + x = 2x$]

In ΔABC

$AC^2 = AB^2 + BC^2$ [By Pythagoras Theorem]

$AC^2 = AB^2 + 9x^2$ ...(*iii*) [$\because BC = BD + DE + EC = 3x$]

Now, multiplying (*i*) by 5, (*ii*) by 8 and (*iii*) by 3, we get

$5AD^2 = 5AB^2 + 5x^2$ ...(*iv*)

$8AE^2 = 8AB^2 + 32x^2$ ...(*v*)

$3AC^2 = 3AB^2 + 27x^2$ ...(*vi*)

Adding (*iv*) and (*vi*), we get

$5AD^2 + 3AC^2 = 5AB^2 + 5x^2 + 3AB^2 + 27x^2$

$5AD^2 + 3AC^2 = 8AB^2 + 32x^2$ ...(*vii*)

Subtracting (*v*) from (*vii*), we get

$5AD^2 + 3AC^2 - 8AE^2 = 8AB^2 + 32x^2 - 8AB^2 - 32x^2$

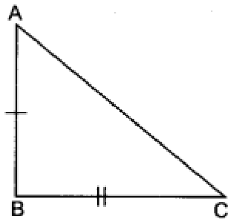
$5AD^2 + 3AC^2 - 8AE^2 = 0$

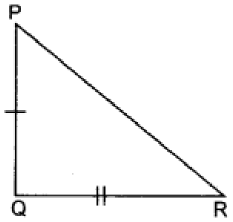
$\therefore$ $5AD^2 + 3AC^2 = 8AE^2$ Hence proved.

Question 39.

Prove that in a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Solution:





Given: ΔABC in which $AC^2 = AB^2 + BC^2$

To prove: $\angle B = 90°$

Construction: Construct a ΔPQR right angled at Q such that PQ = AB and QR = BC.

Proof:

Now, in ΔPQR, we have

$PR^2 = PQ^2 + QR^2$ [By Pythagoras theorem as $\angle Q = 90°$]

or $PR^2 = AB^2 + BC^2$ [By construction] ...(*i*)

But $AC^2 = AB^2 + BC^2$ [Given] ...(*ii*)

So, $AC = PR$ [From (*i*) and (*ii*)] ...(*iii*)

Now, in ΔABC and ΔPQR

$AB = PQ$ [By construction]

$BC = QR$ [By construction]

$AC = PR$ [Proved in (*iii*) above]

So, $\Delta ABC \cong \Delta PQR$ [By SSS]

Therefore, $\angle B = \angle Q$

So, $\angle B = 90°$ [$\because \angle Q = 90°$]

Hence, angle B is right angle.

Question 40.

In ΔABC, AD is the median to BC and in ΔPQR, PM is the median to QR.

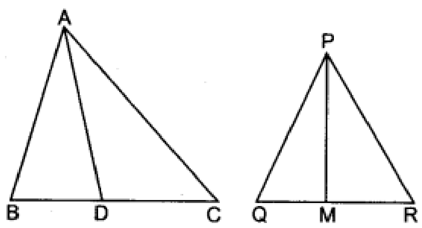
If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$, prove that ΔABC ~ ΔPQR.

Solution:

Given: In ΔABC, AD is the median, i.e. BD = DC

In ΔPQR, PM is the median, i.e. QM = MR

and $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$



To prove: $\Delta ABC \sim \Delta PQR$

Proof: ∵ $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

$\Rightarrow \quad \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$

$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

$\Rightarrow \quad \Delta ABD \sim \Delta PQM$ [By SAS]

$\Rightarrow \quad \angle B = \angle Q$ [Corresponding angles of similar angles]

Now, in ΔABC and ΔPQR

$\frac{AB}{PQ} = \frac{BC}{QR}$ [Given]

$\angle B = \angle Q$ [Proved above]

$\therefore \quad \Delta ABC \sim \Delta PQR$ [By SAS]

2012

Short Answer Type Question I [2 Marks]

Question 41.

In the given figure, if AB || DC, find the value of x.

Solution:

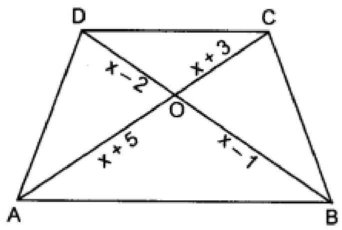
Given: AB || DC ∴ ΔDOC ~ ΔBOA

$\therefore \quad \frac{OD}{OB} = \frac{OC}{OA} \Rightarrow \frac{x-2}{x-1} = \frac{x+3}{x+5}$

$(x-2)(x+5) = (x+3)(x-1)$

$\Rightarrow \quad x^2 + 3x - 10 = x^2 + 2x - 3$

$\Rightarrow \quad x = 7$



Short Answer Type Question II [3 Marks]

Question 42.

In the given figure PQ || BA; PR || CA. If PD = 12 cm. Find BD X CD.

Solution:

In ΔBRD,

BR || PQ

$\therefore \quad \frac{BD}{PD} = \frac{RD}{QD}$

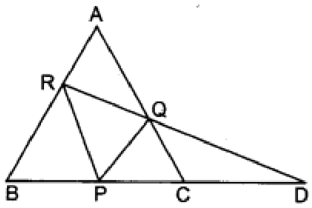
In ΔRDP, $PR \parallel QC$ [Given]

$$\therefore \quad \frac{RD}{QD} = \frac{PD}{CD} \quad ...(ii)$$

From (i) and (ii), we get

$$\frac{PD}{CD} = \frac{BD}{PD}$$

$$\Rightarrow \quad BD \times CD = PD \times PD = 12 \times 12 = 144 \text{ cm}^2$$



2011

Short Answer Type Question I [2 Marks]

Question 43.

If one diagonal of a trapezium divides the other diagonal in the ratio 1 : 3. Prove that one of the parallel sides is three times the other.

Solution:

$$DE : EB = 1:3$$

In ΔAEB and ΔCED, $\angle 1 = \angle 2$ (Alternate angles)

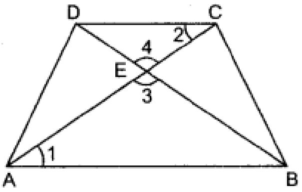
$\angle 3 = \angle 4$ (V.O.A.)

$$\therefore \quad \Delta AEB \sim \Delta CED$$

$$\Rightarrow \quad \frac{AB}{CD} = \frac{BE}{DE}$$

$$\Rightarrow \quad \frac{AB}{CD} = \frac{3}{1} \quad [\because DE : BE = 1:3]$$

$$\Rightarrow \quad AB = 3CD$$

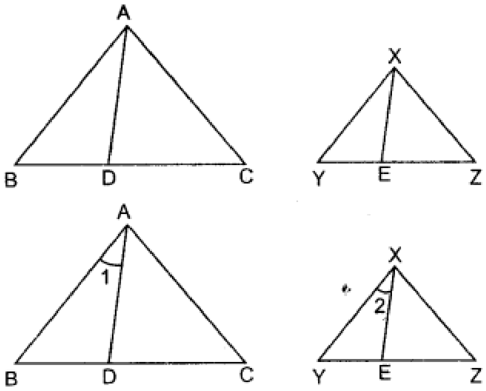


Short Answer Type Questions II [3 Marks]

Question 44.

In given figure ΔABC is similar to ΔXYZ and AD and XE are angle bisectors of ∠A and ∠X respectively such that AD and XE in centimetres are 4 and 3 respectively, find the ratio of area of ΔABD and area of ΔXYE.

Solution:



AD bisects $\angle A$ $\therefore$ $\angle 1 = \frac{1}{2}\angle A$

Similarly $\angle 2 = \frac{1}{2}\angle X$

$\because$ $\Delta ABC \sim \Delta XYZ$

$\therefore$ $\angle A = \angle X$

$\Rightarrow$ $\frac{1}{2}\angle A = \frac{1}{2}\angle X \Rightarrow \angle 1 = \angle 2$

Also $\angle B = \angle Y$

$\therefore$ $\Delta ABD \sim \Delta XYE$

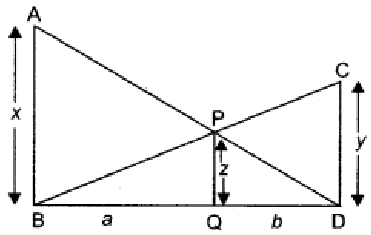
$$\frac{\text{Area } \Delta ABD}{\text{Area } \Delta XYE} = \frac{AD^2}{XE^2} = \frac{4^2}{3^2} = \frac{16}{9}$$

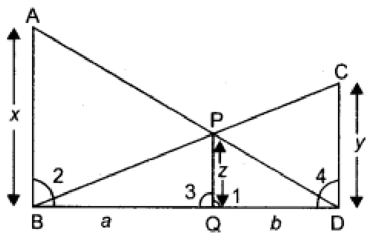
Question 45

In figure, AB || PQ || CD, AB = x units, CD =y units and PQ = z units, prove that

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

Solution:





Let BQ = a units, DQ = b units

In ΔADB and ΔPDQ,

$\because$ PQ || AB $\therefore$ $\angle 1 = \angle 2$,

and $\angle ADB = \angle PDQ$ (Com

$\therefore$ $\Delta ADB \sim \Delta PDQ$

Similarly $\Delta BCD \sim \Delta BPQ$

$\because$ $\Delta ADB \sim \Delta PDQ$

$\therefore$ $\frac{AB}{PQ} = \frac{BD}{DQ}$

$$\frac{x}{z} = \frac{a+b}{b} \Rightarrow \frac{x}{z} = \frac{a}{b} + 1$$

$\Rightarrow$ $\frac{x}{z} - 1 = \frac{a}{b}$

Also, $\Delta BCD \sim \Delta BPQ$

$$\therefore \quad \frac{BD}{BQ} = \frac{CD}{PQ} \Rightarrow \frac{a+b}{a} = \frac{y}{z}$$

$$1 + \frac{b}{a} = \frac{y}{z} \Rightarrow \frac{b}{a} = \frac{y}{z} - 1 \Rightarrow \frac{b}{a} = \frac{y-z}{z}$$

$$\Rightarrow \quad \frac{a}{b} = \frac{z}{y-z} \quad \text{...}(ii)$$

From (*i*) and (*ii*), we get

$$\frac{x}{z} - 1 = \frac{z}{y-z} \Rightarrow \frac{x}{z} = \frac{z}{y-z} + 1$$

$$\frac{x}{z} = \frac{z+y-z}{y-z}$$

$$\frac{x}{z} = \frac{y}{y-z} \Rightarrow \frac{z}{x} = \frac{y-z}{y}$$

$$\frac{z}{x} = 1 - \frac{z}{y}$$

$$z\left(\frac{1}{x}\right) = z\left(\frac{1}{z} - \frac{1}{y}\right)$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{z} - \frac{1}{y} \Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

Hence proved.

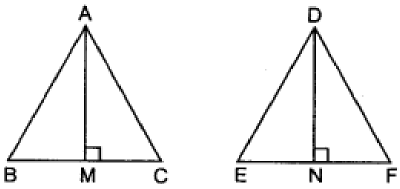
Long Answer Type Questions [4 Marks]

Question 46.

The area of two similar triangles are 49 cm^2 and 64 cm^2 respectively. If the difference of the corresponding altitudes is 10 cm, then find the lengths of altitudes (in centimetres).

Solution:

$\Delta ABC \sim \Delta DEF$ (Given)



$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \quad \frac{49}{64} = \frac{BC^2}{EF^2} \Rightarrow \frac{BC}{EF} = \frac{7}{8}$$

$$\text{Also} \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{\frac{1}{2}BC \times AM}{\frac{1}{2}EF \times DN} \Rightarrow \frac{49}{64} = \frac{BC}{EF} \times \frac{AM}{DN}$$

$$\Rightarrow \quad \frac{49}{64} = \frac{7}{8} \times \frac{AM}{DN} \Rightarrow \frac{7}{8} = \frac{AM}{DN} \Rightarrow DN = \frac{8}{7}AM$$

Also $DN - AM = 10$ (Given)

$$\Rightarrow \quad \frac{8}{7}AM - AM = 10 \Rightarrow \frac{1}{7}AM = 10$$

$$AM = 70 \text{ cm}$$

$$\therefore \quad DN = 80 \text{ cm}$$

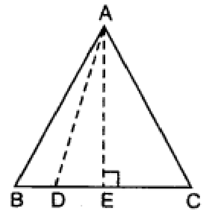
Question 47.

In an equilateral triangle ABC, D is a point on side BC such that 4BD = BC. Prove that ADcm^2 = BCcm^2.

Solution:

In equilateral ΔABC,

$$4BD = BC$$



Construction: Draw $AE \perp BC$.

$\therefore \quad BE = \frac{1}{2}BC.$

In right ΔAED,

$$AD^2 = DE^2 + AE^2$$

$\Rightarrow \quad AE^2 = AD^2 - DE^2 \quad \ldots (i)$

In right ΔAEB,

$$AB^2 = AE^2 + BE^2$$

$\Rightarrow \quad AB^2 = AD^2 - DE^2 + BE^2 \quad$ [Using (i)]

$\Rightarrow \quad AB^2 + DE^2 - BE^2 = AD^2$

$\Rightarrow \quad AB^2 + (BE - BD)^2 - BE^2 = AD^2$

$\Rightarrow \quad AB^2 + BE^2 + BD^2 - 2BE \,.\, BD - BE^2 = AD^2$

$\Rightarrow \quad AB^2 + BD^2 - 2BE \,.\, BD = AD^2$

$\Rightarrow \quad AB^2 + \left(\frac{1}{4}BC\right)^2 - 2 \times \frac{1}{2}BC \times \frac{1}{4}BC = AD^2$

$\Rightarrow \quad AB^2 + \frac{1}{16}BC^2 - \frac{1}{4}BC^2 = AD^2$

$\Rightarrow \quad BC^2 - \frac{3}{16}BC^2 = AD^2 \quad (\because AB = BC)$

$\Rightarrow \quad \frac{13BC^2}{16} = AD^2$

$\Rightarrow \quad 13\,BC^2 = 16\,AD^2$

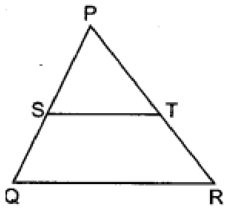
2010

Very Short Answer Type Questions [1 Mark]

Question 48.

In figure, S and T are points on the sides PQ and PR, respectively of APQR, such that PT = 2 cm, TR = 4 cm and ST is parallel to QR. Find the ratio of the areas of ΔPST and ΔPQR.

Solution:



S and T are points on the sides PQ and PR of ΔPQR and PT = 2 cm, TR = 4 cm, ST || QR

In ΔPST and ΔPQR

$\angle S = \angle Q \quad$ [Corresponding angles]

$\angle P = \angle P$

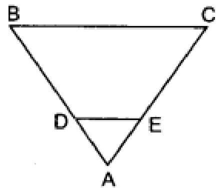
$\therefore \quad \Delta PST \sim \Delta PQR$

$\therefore \quad \frac{ar(\Delta PST)}{ar(\Delta PQR)} = \frac{PT^2}{PR^2} = \frac{2^2}{6^2} = \frac{4}{36} = \frac{1}{9}$

Question 49.

In figure, DE || BC in AABC such that that BC = 8 cm, AB = 6 cm and DA = 1.5 cm. Find DE.

Solution:



DE || BC (Given)

In ΔADE and ΔABC,

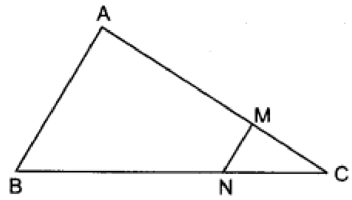
$$\angle ADE = \angle ABC \quad \text{[Corresponding angles]}$$
$$\angle A = \angle A \quad \text{[Common]}$$
$$\therefore \quad \Delta ADE \sim \Delta ABC \quad \text{[AA similarity]}$$
$$\Rightarrow \quad \frac{AD}{AB} = \frac{DE}{BC}$$

Now,
$$\frac{1.5}{6} = \frac{DE}{8}$$
$$DE = \frac{1.5 \times 8}{6} = 2 \text{ cm}$$

Question 50.

In figure, MN || AB, BC = 7.5 cm, AM = 4 cm and MC = 2 cm. Find the length BN.

Solution:



In ΔABC, $MN \parallel AB \Rightarrow \Delta ABC \sim \Delta MNC$

$$\Rightarrow \quad \frac{MC}{AM} = \frac{NC}{BN}$$
$$\Rightarrow \quad \frac{2}{4} = \frac{7.5 - x}{x} \quad [\text{Let } x = BN]$$
$$\Rightarrow \quad x = 15 - 2x$$
$$\Rightarrow \quad 3x = 15 \Rightarrow x = 5$$

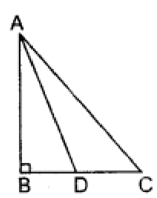
Hence, $BN = 5$ cm.

Short Answer Type Questions I [2 Marks]

Question 51.

Triangle ABC is right angled at B, and D is mid-point of BC. Prove that $AC^2 = 4AD^2 - 3AB^2$.

Solution:



Given: ΔABC with $\angle B = 90°$

D is the mid-point of BC.

To prove: $AC^2 = 4AD^2 - 3AB^2$

Proof: In ΔABC, $\angle B = 90°$ [Given]

$$AC^2 = AB^2 + BC^2 \quad \text{[By Pythagoras theorem]}$$
$$= AB^2 + (2BD)^2$$
$$= AB^2 + 4BD^2 \quad ...(i)$$

In ΔABD, $AD^2 = AB^2 + BD^2$ [Using Pythagoras theorem]

$$\Rightarrow \quad BD^2 = AD^2 - AB^2 \quad ...(ii)$$

From (*i*) and (*ii*), we get

$$AC^2 = AB^2 + 4(AD^2 - AB^2) = AB^2 + 4AD^2 - 4AB^2$$
$$AC^2 = 4AD^2 - 3AB^2$$

Hence proved.

Question 52.

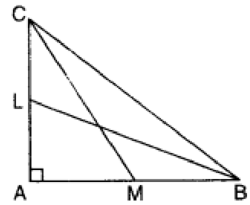
If BL and CM are medians of a triangle ABC right angled at A, then prove that $4(BL^2 + CM^2) =$

$5BC^2$.

Solution:

Given: A right angled triangle ABC, right angled at A.
BL and CM are the medians.

To prove: $4(BL^2 + CM^2) = 5BC^2$



Proof: In right angled triangle CAB,

$$BC^2 = AC^2 + AB^2 \quad \text{[By Pythagoras theorem] ...(i)}$$

In right-angled triangle CAM,

$$CM^2 = AC^2 + AM^2$$

Also, $$AM = \frac{1}{2}AB \quad \text{[As CM is median]}$$

$\therefore$ $$CM^2 = AC^2 + \left(\frac{1}{2}AB\right)^2$$

$\Rightarrow$ $$4CM^2 = 4AC^2 + AB^2 \quad ...(ii)$$

In right-angled triangle LAB,

$$BL^2 = AL^2 + AB^2$$

Also, $$AL = \frac{1}{2}AC \quad \text{[As BL is median]}$$

$\therefore$ $$BL^2 = \left(\frac{1}{2}AC\right)^2 + AB^2$$

$\Rightarrow$ $$4BL^2 = AC^2 + 4AB^2 \quad ...(iii)$$

Adding (*ii*) and (*iii*), we get

$$4BL^2 + 4CM^2 = 5AC^2 + 5AB^2$$

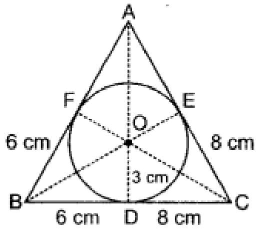
$\Rightarrow$ $$4(BL^2 + CM^2) = 5(AC^2 + AB^2)$$

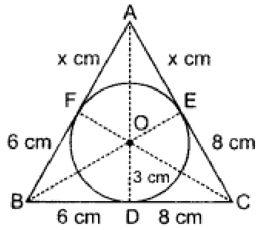
$\Rightarrow$ $$4(BL^2 + CM^2) = 5BC^2 \quad \text{[From (i)] Hence proved.}$$

Question 53.

In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 6 cm and 8 cm respectively. Find the side AB if the area of ΔABC = 63 cm2.

Solution:





Area of ΔABC = area of ΔOBC + area of ΔOAB + area of ΔOAC

$\Rightarrow$ $$63 = \frac{1}{2} \times 14 \times 3 + \frac{1}{2} \times (6 + x) \times 3 + \frac{1}{2} \times (8 \times x) \times 3$$

$\Rightarrow$ $$63 = \frac{3}{2}(14 + 6 + x + 8 + x)$$

$\Rightarrow$ $$42 = 28 + 2x$$

$\Rightarrow$ $$2x = 14 \Rightarrow x = 7$$

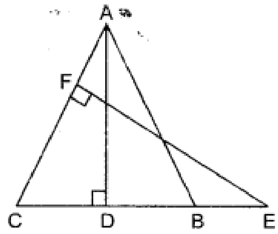
$\therefore$ $$AB = (6 + 7) \text{ cm} = 13 \text{ cm.}$$

Short Answer Type Question II [3 Marks]

Question 54.

In figure, ABC is an isosceles triangle in which AB = AC. E is a point on the side CB produced, such that FE ⊥ AC. If AD ⊥ CB, prove that AB X EF = AD X EC.

Solution:



In ΔADB and ΔEFC,

$\angle D = \angle F$ [Each 90°]

and $\angle B = \angle C$

[Angles opp. to equal sides of a triangle are equal]

$\Rightarrow \Delta ABD \sim \Delta ECF$ [AA similarity]

$\therefore \dfrac{AB}{EC} = \dfrac{AD}{EF}$ [Corresponding

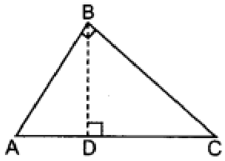
$\therefore AB \times EF = AD \times EC$

Long Answer Type Questions [4 Marks]

Question 55.

Prove that in a right angle triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides. Point D is the mid-point of the side BC of a right triangle ABC, right angled at C. Prove that, $4AD^2 = 4AC^2 + BC^2$..

Solution:



Given: In ΔABC, $\angle B = 90°$

To prove: $AC^2 = AB^2 + BC^2$

Construction: Draw $BD \perp AC$

Proof: Since, in ΔABC, $\angle B = 90°$ and $BD \perp AC$

so, $\Delta ADB \sim \Delta ABC$ [If a $\perp$ is drawn from the vertex of the rt. angle of rt. Δ to the hypotenuse then Δ's on both sides of the $\perp$ are similar to the whole Δ and to each other]

and $\Delta BDC \sim \Delta ABC$

Now, $\Delta ADB \sim \Delta ABC$

$$\Rightarrow \quad \frac{AD}{AB} = \frac{AB}{AC} \Rightarrow AB^2 = AC.AD \quad \text{...}(i)$$

Again, $\Delta BDC \sim \Delta ABC$

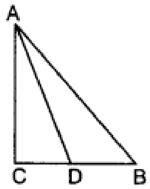
$$\Rightarrow \quad \frac{CD}{BC} = \frac{BC}{AC} \Rightarrow BC^2 = AC.CD \quad \text{...}(ii)$$

Adding equation (*i*) and (*ii*), we get

$$\begin{aligned} AB^2 + BC^2 &= AC\cdot AD + AC\cdot CD \\ &= AC(AD + CD) \\ &= AC\cdot AC \\ &= AC^2 \end{aligned}$$

$\therefore \quad AB^2 + BC^2 = AC^2$ Hence proved

Other part:



In ΔADC, $\angle C = 90°$

$\therefore \quad AD^2 = AC^2 + CD^2$ ...(*i*)

D is mid point of BC

$\therefore \quad CD = \frac{1}{2}BC$

Putting this value in equation (*i*), we get

$$AD^2 = AC^2 + \left(\frac{BC}{2}\right)^2$$

$$AD^2 = AC^2 + \frac{BC^2}{4}$$

$$4AD^2 = 4AC^2 + BC^2$$

Question 56.

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Using the above, prove the following:

If the areas of two similar triangles are equal, then prove that the triangles are congruent.

Solution:

To prove: $\Delta ABC \cong \Delta PQR$

Proof: Using the above result, we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{AC^2}{PR^2} = \frac{BC^2}{QR^2}$$

Also $ar(\Delta ABC) = ar(\Delta PQR)$ [Given]

$$\therefore \quad 1 = \frac{AB^2}{PQ^2} = \frac{AC^2}{PR^2} = \frac{BC^2}{QR^2}$$

$\Rightarrow \quad AB = PQ, AC = PR, BC = QR$

$\Rightarrow \quad \Delta ABC \cong \Delta PQR$ [SSS]

Question 57.

In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite the first side is a right angle.

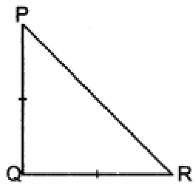
Using the above, do the following:

In an isosceles triangle PQR, PQ = QR and $PR^2 = 2PQ^2$. . Prove that ZQ is a right angle.

Solution:

Given: In isosceles ΔPQR, $PQ = QR$ and $PR^2 = 2PQ^2$.

To prove: $\angle Q$ is a right angle.



Proof: $PR^2 = 2PQ^2$ [Given]

$= PQ^2 + PQ^2$

$\Rightarrow PR^2 = PQ^2 + QR^2$ [$\because PQ = QR$]

$\Rightarrow \Delta PQR$ is right angled at Q [Using above result]

$\therefore \angle Q$ is a right angle.

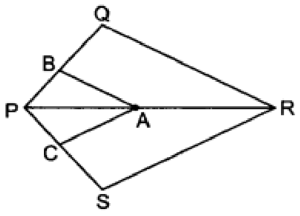
Question 58.

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.

Using the above, do the following:

In figure, BA || QR, and CA || SR, prove

Solution:



In ΔPQR and ΔPSR

$QR \parallel BA$

$\therefore$ By BPT $\frac{PB}{BQ} = \frac{PA}{AR}$ ...(*i*)

In ΔPSR $CA \parallel SR$

$\therefore \frac{PC}{CS} = \frac{PA}{AR}$ [by BPT] ...(*ii*)

Equating (*i*) and (*ii*)

$$\frac{PB}{BQ} = \frac{PC}{CS}$$

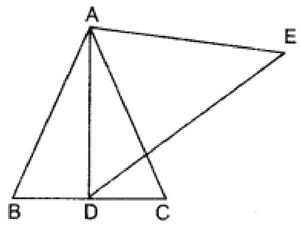
$$\Rightarrow \frac{QB}{BP} = \frac{SC}{CP}$$

Question 59.

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides. Using the above, do the following:

AD is an altitude of an equilateral triangle ABC. On AD as base, another equilateral triangle ADE is constructed. Prove that, area (ΔADE): area (ΔABC) =3:4

Solution:



ΔABC and ΔADE are equilateral triangles.

$$\frac{ar\,\Delta ABC}{ar\,\Delta ADE} = \frac{(AB)^2}{(AD)^2} \qquad ...(i)$$

where $\qquad AD = \frac{\sqrt{3}}{2}AB$

Putting in equation (i)

$$\frac{ar\,\Delta ABC}{ar\,\Delta ADE} = \left[\frac{(AB)^2}{\left(\frac{\sqrt{3}}{2}AB\right)^2}\right] = \left(\frac{2}{\sqrt{3}}\cdot\frac{AB}{AB}\right)^2$$

$$\frac{ar\,\Delta ABC}{ar\,\Delta ADE} = \frac{4}{3}$$

Question 60.

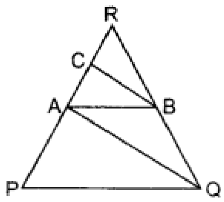
If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.

Using the above, do the following:

In figure, PQ || AB and AQ || CB.

Prove that AR^2 = PR. CR.

Solution:



$$\Rightarrow \qquad \frac{AR}{PR} = \frac{RB}{RQ} \qquad \text{(Using above result) ...(i)}$$

In ΔAQR, BC || AQ

$$\Rightarrow \qquad \frac{CR}{AR} = \frac{RB}{RQ} \qquad \text{(Using above result) ...(ii)}$$

From (i) and (ii), we have

$$\frac{AR}{PR} = \frac{CR}{AR} \Rightarrow AR^2 = PR \cdot CR$$

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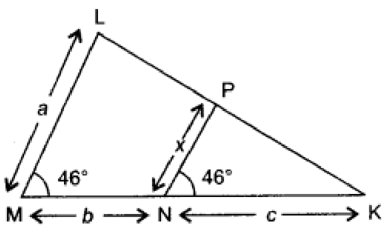
Very Short Answer Type Questions [1 Mark]

Question 61.

In fig. ∠M =∠N = 46°, express JC in terms of a, b andc, where a, andc are lengths ofLM, MN and NK respectively.

Solution:



In ΔLMK and ΔPNK

$$\angle M = \angle N = 46^\circ \quad \text{[Given]}$$
$$\angle K = \angle K \quad \text{[Common]}$$
$$\Delta LMK \sim \Delta PNK \quad \text{[By AA similarity]}$$
$$\therefore \quad \frac{ML}{NP} = \frac{MK}{NK}$$
$$\frac{a}{x} = \frac{b+c}{c} \Rightarrow x = \frac{ac}{b+c}$$

Question 62.

If the areas of two similar triangles are in ratio 25 : 64, write the ratio of their corresponding sides.

Solution:

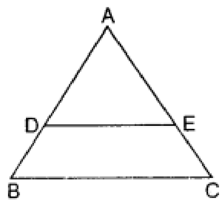
$$\frac{\text{ar of triangle I}}{\text{ar of triangle II}} = \left(\frac{\text{Corresponding side of triangle I}}{\text{Corresponding side of triangle II}}\right)^2$$
$$\Rightarrow \quad \frac{25}{64} = \left(\frac{\text{Side of triangle I}}{\text{Side of triangle II}}\right)^2 \Rightarrow \frac{\text{Side of triangle I}}{\text{Side of triangle II}} = \frac{5}{8}$$

Question 63.

In a ΔABC, DE || BC. If DE = – BC and area of ΔABC = 81 cm2, find the area of ΔADE.

Solution:

In ΔABC and ΔADE



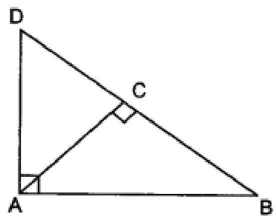
$$\angle A = \angle A \quad \text{[Common]}$$
$$\angle B = \angle D \quad \text{[DE || BC]}$$
$$\Delta ABC \sim \Delta ADE \quad \text{[By AA similarity]}$$
$$\frac{ar\,\Delta ABC}{ar\,\Delta ADE} = \left(\frac{BC}{DE}\right)^2 = \left(\frac{3}{2} \cdot \frac{DE}{DE}\right)^2 = \frac{9}{4}$$
$$\frac{ar\,\Delta ABC}{ar\,\Delta ADE} = \frac{9}{4}$$
$$\Rightarrow \quad \frac{81}{ar\,\Delta ADE} = \frac{9}{4}$$
$$36\text{ cm}^2 = ar\,\Delta ADE$$
$$\Rightarrow \quad ar\,\Delta ADE = 36\text{ cm}^2$$

Short Answer Type Questions I [2 Marks]

Question 64.

In figure,ΔABD is a right triangle, right angled at A and AC⊥BD. Prove that AB^2 = BC.BD.

Solution:



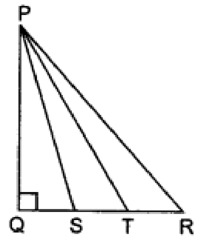
In ΔBAD and ΔBCA

$$\angle B = \angle B \quad \text{[Common]}$$
$$\angle BAD = \angle BCA \quad [90^\circ \text{ each}]$$
$$\Delta BAD \sim \Delta BCA \quad \text{[By AA similarity]}$$
$$\therefore \quad \frac{BA}{BC} = \frac{BD}{BA}$$
$$(BA)^2 = BD.BC$$
$$(AB)^2 = BC.BD$$

Question 65.

In figure, S and T trisect the side QR of a right triangle PQR.Prove that $8PT^2 = 3PR^2 + 5PS^2$.

Solution:



Given: Right triangle PQR. S and T trisect QR.

To prove: $8PT^2 = 3PR^2 + 5PS^2$

Proof: $QS = ST = TR = \frac{1}{3}QR$

In right triangle PQS,

$$PS^2 = PQ^2 + QS^2$$

In right triangle PQT,

$$PT^2 = PQ^2 + QT^2$$

In right triangle PQR,

$$PR^2 = PQ^2 + QR^2$$

Subtracting (*iii*) from (*ii*), we get

$$PS^2 - PT^2 = QS^2 - QT^2$$
$$\Rightarrow \quad PS^2 - PT^2 = \left(\frac{1}{3}QR\right)^2 - \left(\frac{2}{3}QR\right)^2$$
$$= \frac{1}{9}QR^2 - \frac{4}{9}QR^2 = \frac{-1}{3}QR^2$$
$$\Rightarrow \quad 3PS^2 - 3PT^2 = -QR^2$$

Subtracting (*iv*) from (*iii*), we get

$$PT^2 - PR^2 = QT^2 - QR^2 = \left(\frac{2}{3}QR\right)^2 - QR^2$$
$$\Rightarrow \quad PT^2 - PR^2 = \frac{4}{9}QR^2 - QR^2 = \frac{-5}{9}QR^2$$
$$\Rightarrow \quad 9PT^2 - 9PR^2 = -5QR^2$$

Substituting for $(-QR^2)$ from (*v*) in (*vi*), we get

$$\Rightarrow \quad 9PT^2 - 9PR^2 = 5(3PS^2 - 3PT^2)$$
$$\Rightarrow \quad 9PT^2 - 9PR^2 = 15PS^2 - 15PT^2$$
$$\Rightarrow \quad 24PT^2 = 15PS^2 + 9PR^2$$
$$\Rightarrow \quad 8PT^2 = 5PS^2 + 3PR^2$$

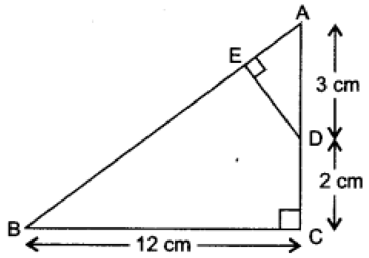
Hence proved.

Short Answer Type Questions II [3 Marks]

Question 66.

In figure, AABC is right angled at C and DE ⊥ AB. Prove that ΔABC ~ ΔADE and hence find the lengths of AE and DE.

Solution:



Given: ΔABC and ΔADE right angled at C and E.

Proof: In ΔABC and ΔADE

$$\angle C = \angle E \quad \text{[90° Each]}$$
$$\angle A = \angle A \quad \text{[Common angle]}$$
$$\Delta ABC \sim \Delta ADE \quad \text{[By AA similarity]}$$

Since, $\Delta ABC \sim \Delta ADE$

In ΔABC, $AB^2 = AC^2 + BC^2$

$$AB^2 = 25 + 144 = 169$$

$\Rightarrow$ $AB = 13$

Now, $\dfrac{AB}{AD} = \dfrac{BC}{DE} = \dfrac{AC}{AE}$

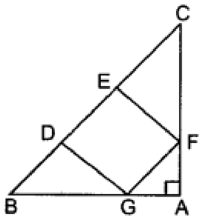
$$\frac{13}{3} = \frac{12}{DE} = \frac{5}{AE}$$

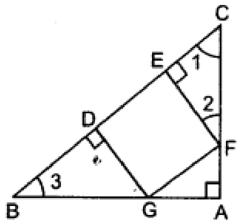
Then, $AE = \dfrac{15}{13}, DE = \dfrac{36}{13}$

Question 67.

In figure, DEFG is a square and $\angle BAC = 90°$. Show that $DE^2 = BD \times EC$.

Solution:





Given: DEFG is a square and $\angle BAC = 90°$.

To prove: $DE^2 = BD \times EC$

Proof: In ΔBDG and ΔCEF

$$\angle 1 + \angle 3 = 90° \quad \text{[Because } \angle A = 90°\text{]}$$
$$\angle 1 + \angle 2 = 90° \quad \text{[Because } \angle CEF = 90°\text{]}$$
$$\angle 1 + \angle 2 = \angle 1 + \angle 3$$

So, $\angle 2 = \angle 3$

and $\angle CEF = \angle BDG$ [90° each]

Hence, $\Delta BDG \sim \Delta FEC$ [AA Similarity]

$\therefore$ $\dfrac{BD}{FE} = \dfrac{DG}{EC} = \dfrac{BG}{FC}$

Now, DEFG is a square

$\therefore$ $DE = EF = FG = DG$

Then, $\dfrac{BD}{DE} = \dfrac{DE}{EC}$

$$DE^2 = BD . EC.$$

[

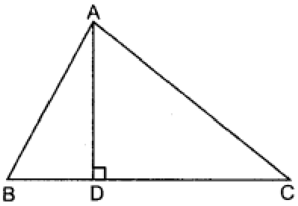
Question 68.

In figure, AD ⊥ BC and BD = 1/3 CD.
Prove that $2CA^2 = 2AB^2 + BC^2$.

Solution:

In figure, AD ⊥ BC and BD = $\frac{1}{3}$CD.

Prove that $2CA^2 = 2AB^2 + BC^2$. [All India]



Given: In ΔABC, AD ⊥ BC and BD = $\frac{1}{3}$CD

To prove: $2CA^2 = 2AB^2 + BC^2$

Proof: ∵ BC = BD + CD and BD = $\frac{1}{3}$CD [Given]

$$\therefore \quad BC = \frac{1}{3}CD + CD = \frac{4}{3}CD$$

$$\Rightarrow \quad CD = \frac{3}{4}BC \qquad ...(i)$$

In right angled ΔADC,

$$AC^2 = CD^2 + AD^2 \qquad \text{[By Pythagoras theorem] ...(ii)}$$

In right angled ΔADB,

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow \quad AD^2 = AB^2 - BD^2$$

Substituting in (*ii*), we get

$$\Rightarrow \quad AC^2 = CD^2 + AB^2 - BD^2$$

$$\Rightarrow \quad AC^2 = CD^2 + AB^2 - \left(\frac{1}{3}CD\right)^2 \qquad \left[\text{Put } BD = \frac{1}{3}CD\right]$$

$$\Rightarrow \quad AC^2 = CD^2 - \frac{1}{9}CD^2 + AB^2$$

$$\Rightarrow \quad AC^2 = \frac{8}{9}CD^2 + AB^2$$

$$\Rightarrow \quad AC^2 = \frac{8}{9}\left(\frac{3}{4}BC\right)^2 + AB^2 \qquad \text{[Using (i)]}$$

$$\Rightarrow \quad AC^2 = \frac{8}{9} \times \frac{9}{16} BC^2 + AB^2$$

$$\Rightarrow \quad AC^2 = \frac{1}{2}BC^2 + AB^2$$

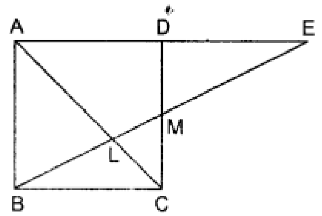
$$\Rightarrow \quad 2AC^2 = BC^2 + 2AB^2$$

Hence proved.

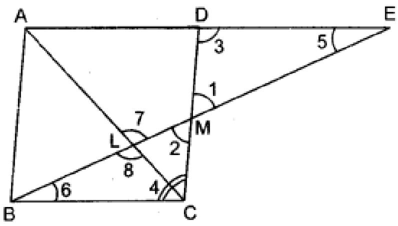
Question 69.

In figure, M is mid-point of side CD of a parallelogram ABCD. The line BM is drawn intersecting AC at L and AD produced at E. Prove that EL = 2BL.

Solution:



Proof: In ΔMDE and ΔMCB



$DM = CM$ [Given]

$\angle 1 = \angle 2$ [Vertically opposite]

$\angle 3 = \angle 4$ [$\because$ BC || AE and DC is a transversal]

[Alt. int $\angle s$]

$\therefore \Delta MDE \cong \Delta MCB$ [ASA Congruency]

$DE = BC$ [CPCT] ... (*i*)

Also $BC = AD$ ... (*ii*)

(Opposite sides of the parallelogram)

$\therefore$ $AD = DE$ [On equating (*i*) and (*ii*)]

Now, $AE = AD + DE$

$\Rightarrow$ $AE = 2AD$ [Put DE = AD]

In ΔBLC and ΔELA,

$\angle 5 = \angle 6$ [Alt. int. angles]

and $\angle 7 = \angle 8$ [Vertically opposite angles]

$\therefore$ $\Delta BLC \sim \Delta ELA$ [AA similarity]

$$\Rightarrow \frac{BL}{EL} = \frac{LC}{LA} = \frac{BC}{AE} \Rightarrow \frac{BL}{EL} = \frac{BC}{AE} \Rightarrow \frac{BL}{EL} = \frac{BC}{2AD}$$

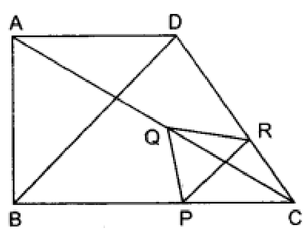
$$\Rightarrow \frac{BL}{EL} = \frac{AD}{2AD} \quad [\because BC = AD]$$

$$\Rightarrow \frac{BL}{EL} = \frac{1}{2} \Rightarrow EL = 2BL$$

Question 70.

In figure, two triangles ABC and DBC lie on the same side of base BC. P is a point on BC such that PQ || BA and PR || BD. Prove that QR || AD.

Solution:



Given: In ΔABC, PQ || AB and PR || BD

To prove: QR || AD

Proof: In ΔABC, PQ || AB

$\Rightarrow$ By BPT, $\frac{CP}{BP} = \frac{CQ}{AQ}$

Now in ΔBCD, PR || BD

$\Rightarrow$ By using BPT

$$\frac{CP}{BP} = \frac{CR}{RD}$$

From (*i*) and (*ii*), we get

$$\frac{CQ}{AQ} = \frac{CR}{RD}$$

$\Rightarrow$ By converse of BPT, QR || AD

Long Answer Type Questions [4 Marks]

Question 71.

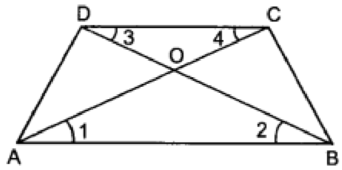
Prove that the ratio of areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Using the above, do the following:

In a trapezium ABCD, AC and BD intersecting at O, AB|| DC and AB = 2CD, if area of ΔAOB

= 84 cm2, find the area of ΔCOD

Solution:



Given: ABCD is a trapezium

$$AB \parallel CD$$

Also $AB = 2CD$

To find: Area of ΔCOD

Now, in ΔAOB and ΔCOD

$$\angle 1 = \angle 4$$
$$\angle 2 = \angle 3$$

$\therefore$ $\Delta AOB \sim \Delta COD$

$$\frac{ar\Delta AOB}{ar\Delta COD} = \frac{(AB)^2}{(CD)^2}$$

$$\frac{84}{ar\Delta COD} = \frac{(2CD)^2}{(CD)^2}$$

$$\frac{84}{ar\Delta COD} = \frac{4(CD)^2}{(CD)^2}$$

$$\frac{84}{ar\Delta COD} = \frac{4}{1}$$

$$ar\ \Delta COD = 21\ cm^2$$

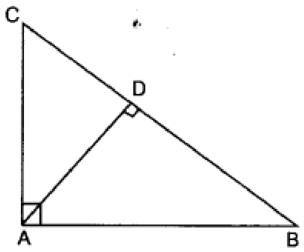
Question 72.

Prove that in a right angle triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides. ‘

Using the above, do the following:

Prove that in a ΔABC, if AD is perpendicular to BC, then $AB^2 + CD^2 = AC^2 + BD^2$.

Solution:



In ΔABC, AD ⊥ BC

To prove: $AB^2 + CD^2 = AC^2 + BD^2$

Proof: In ΔABD, $\angle D = 90°$

$$AB^2 = BD^2 + AD^2$$

[By Pythagoras theorem]

$\therefore$ $AD^2 = AB^2 - BD^2$ ...(*i*)

In ΔADC, ∠D = 90°

$\therefore$ $AD^2 = AC^2 - CD^2$ ...(*ii*) [By Pythagoras theorem]

On equating equation (*i*) and (*ii*), we get

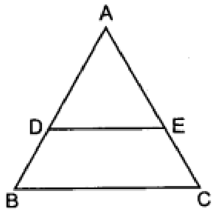
$$AB^2 - BD^2 = AC^2 - CD^2$$
$$AB^2 + CD^2 = AC^2 + BD^2$$

Question 73.

Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided is the same ratio.

Using the above result, do the following:
In figure, DE|| BC and BD = CE. Prove that ΔABC is an isosceles triangle.

Solution:



$\because$ BD = CE [Given]

So, AD = AE $\left\{\because \frac{AD}{DB} = \frac{AE}{EC} \text{ Proved above}\right\}$

$\therefore$ AD + BD = AE + CE

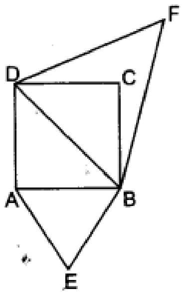
$\Rightarrow$ AB = AC

$\therefore$ ΔABC is an isosceles triangle.

Question 74.

Prove that the ratio of areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.
Using the above, prove the following: The areas of the equilateral triangle described on the side of a square is half the area of the equilateral triangle described, on its diagonal.

Solution:



Equilateral triangle ABE is described on side AB of square ABCD.
Equilateral trianlge BDF is described on diagonal BD of square ABCD.

$\therefore$ $\Delta ABE \sim \Delta BDF$

$$\frac{ar\,\Delta ABE}{ar\,\Delta BDF} = \frac{(AB)^2}{(BD)^2} = \frac{(AB)^2}{(\sqrt{2}AB)^2} \quad [\text{as } BD = \sqrt{2AB}]$$

$$\frac{ar\,\Delta ABE}{ar\,\Delta BDF} = \frac{AB^2}{2AB^2} = \frac{1}{2}$$

$$(ar\,\Delta ABE) = \frac{1}{2}(ar\,\Delta BDF)$$

Question 75.

AABC is an isosceles triangle in which AC = BC. If $AB^2 = 2AC^2$ then, prove that ΔABC is right triangle.

Solution:

$AB^2 = 2AC^2$ [Given]

$\therefore$ $AB^2 = AC^2 + AC^2$

Also AC = BC [Given]

$\therefore$ $AB^2 = BC^2 + AC^2$

By converse of Pythagoras theorem, ΔABC is a right-angled triangle where ∠C = 90°.